JOINT INTERNATIONAL CONFERENCES
10TH EDITION EUROPEAN INTEGRATION REALITIES AND PERSPECTIVES
5TH EDITION THE GLOBAL ADVANCEMENT OF UNIVERSITIES AND COLLEGES

The Stackelberg Model for a Leader of Production and Many Satellites

Catalin Angelo Ioan[1], Gina Ioan[2]

Abstract: Oligopoly is a market situation where there are a small number of bidders (at least two) of a good non-substituent and a sufficient number of consumers. The paper analyses the Stackelberg model for a leader of production and many satellites. There are obtained the equilibrium productions, maximum profits and sales price where one of the company is the leader of quantity, and other satellites. There are also survey the situations where the firm based on its marginal cost of production can effectively take the lead of production.

Keywords: oligopoly; Stackelberg; equilibrium

JEL Classification: D43

1 Introduction

Oligopoly is a market situation where there are a small number of bidders (at least two) of a good non-substituent and a sufficient number of consumers. Oligopoly composed of two producers called duopoly.

Considering any number of competing firms producing the same normal good, is interesting the analysis of each activity in response to the activity of other companies.

Each of them when it sets the production and the sale price will be considered the productions and prices of other firms. If one of the companies will settle the price or the quantity produced, the other adjusting after it, then the price will be called leader of price or leader of production, the others called price satellites or production satellites.

2 The Stackelberg Model for a Leader of Production and Many Satellites

Consider now m firms F_i, $i=\overline{1,m}$, a function of price: $p(Q)=a-bQ$, $a,b>0$ and the total cost of production $TC_i=\alpha_i Q$, $i=\overline{1,m}$, where α_i is the marginal cost of firm F_i.

[1] Associate Professor, PhD, "Danubius" University of Galati, Romania, Address: 3 Galati Boulevard, 800654 Galati, Romania, Tel.: +40.372.361.102, fax: +40.372.361.290, Corresponding author: catalin_angelo_ioan@univ-danubius.ro.

[2] Assistant professor, PhD, "Danubius" University of Galati, Romania, Address: 3 Galati Boulevard, 800654 Galati, Romania, Tel.: +40.372.361.102, fax: +40.372.361.290, E-mail: ginaioan@univ-danubius.ro.

Consider now that the company F_s is a leader of production, where s=$\overline{1,m}$ is fixed. It sets at a given time a production Q_s.

Considering a certain satellite F_k, it will establish a production that will seek to maximize its profit but at the same time it will take into account the production of the leader which will influence the selling price of the product. Therefore, let the firm F_k, k=$\overline{1,m}$, k≠s reaction function:

$$Q_k = f_k(Q_s),\ i=\overline{1,m},\ i\neq s$$

The leader's profit F_s is through price, a function of both its production and those of satellites:

$$\Pi_s(Q_1,...,Q_m) = p\left(\sum_{i=1}^{m} Q_i\right)Q_s - \alpha_s Q_s$$

Taking into account the reaction functions of satellites: $Q_i = f_i(Q_s)$, i=$\overline{1,m}$, i≠s follows:

$$\Pi_s(Q_s) = p\left(Q_s + \sum_{\substack{i=1\\ i\neq s}}^{m} f_i(Q_s)\right)Q_s - \alpha_s Q_s$$

The profit of a satellite F_k, k≠s is:

$$\Pi_k(Q_1,...,Q_m) = p\left(\sum_{i=1}^{m} Q_i\right)Q_k - \alpha_k Q_k,\ k\neq s$$

The condition of profit maximization of the leader F_s is:

$$\frac{\partial \Pi_s(Q_s)}{\partial Q_s} = a - b\left(2Q_s + Q_s\sum_{\substack{i=1\\ i\neq s}}^{m} f_i'(Q_s) + \sum_{\substack{i=1\\ i\neq s}}^{m} f_i(Q_s)\right) - \alpha_s = 0$$

and for satellite F_k:

$$\frac{\partial \Pi_k(Q_1,...,Q_m)}{\partial Q_k} = a - b\left(Q_k + \sum_{i=1}^{m} Q_i\right) - \alpha_k = 0,\ k=\overline{1,m},\ k\neq s$$

In conditions that the production of the company F_s is given, we have the system of equations:

$$Q_k + \sum_{\substack{i=1\\ i\neq s}}^{m} Q_i = \frac{a-\alpha_k}{b} - Q_s,\ k=\overline{1,m},\ k\neq s$$

Adding the resulting m-1 relationships: $\sum_{\substack{i=1\\ i\neq s}}^{m} Q_i + (m-1)\sum_{\substack{i=1\\ i\neq s}}^{m} Q_i = \sum_{\substack{k=1\\ k\neq s}}^{m} \frac{a-\alpha_k}{b} - (m-1)Q_s$ from where:

$$\sum_{\substack{i=1\\ i\neq s}}^{m} Q_i = \frac{(m-1)a - \sum_{\substack{k=1\\ k\neq s}}^{m} \alpha_k}{mb} - \frac{m-1}{m}Q_s.$$

Substituting in the above formula, we find that:

$$Q_k = f_k(Q_s) = \frac{a - m\alpha_k + \sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i}{mb} - \frac{1}{m}Q_s$$ - the reaction function of F_k to F_s, $k=\overline{1,m}$, $k\neq s$

Substituting in the condition of profit maximization of the leader, follows:

$$Q_s^* = \frac{a - m\alpha_s + \sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i}{2b}$$

the satellite production of F_k being:

$$Q_k^* = \frac{a + m(\alpha_s - 2\alpha_k) + \sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i}{2mb},\ k=\overline{1,m},\ k\neq s$$

But now, the condition that F_s be really production leader returns to: $Q_s^* > Q_k^*$, $k=\overline{1,m}$, $k\neq s$ that is:

$$\frac{a - m\alpha_s + \sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i}{2b} > \frac{a + m(\alpha_s - 2\alpha_k) + \sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i}{2mb},\ k=\overline{1,m},\ k\neq s$$

which finally leads to:

$$\alpha_s < \min_{\substack{k=\overline{1,m}\\ k\neq s}} \frac{(m-1)a + (m-1)\sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i + 2m\alpha_k}{m(m+1)}$$

Therefore, the marginal cost of F_s will be higher limited to:

$$\sup\alpha s = \min_{\substack{k=\overline{1,m}\\ k\neq s}} \frac{(m-1)a + (m-1)\sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i + 2m\alpha_k}{m(m+1)}$$

If $\alpha_i \leq c$ - constant, $i=\overline{1,m}$, $i\neq s$ resulting from the above: $\alpha_s < \frac{(m-1)a + (m^2+1)c}{m(m+1)}$.

Following these considerations, in order that the company F_s to maintain or to assume leadership role it is necessary that its marginal cost is higher limited by supαs.

The maximum profits of the firms will be:

$$\Pi_s^*(Q_1^*,\ldots,Q_m^*) = p\left(\sum_{i=1}^{m}Q_i^*\right)Q_s^* - \alpha_s Q_s^* = \frac{\left(a - m\alpha_s + \sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i\right)^2}{4mb}$$

$$\Pi_k^*\left(Q_1^*,\ldots,Q_m^*\right)=p\left(\sum_{i=1}^{m}Q_i^*\right)Q_k^*-\alpha_k Q_k^*=\frac{\left(a+m\left(\alpha_s-2\alpha_k\right)+\sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i\right)^2}{4m^2b},\ k=\overline{1,m},\ k\neq s$$

The condition that the profit leader be higher than that of any satellite returns to:

$$\frac{\left(a-m\alpha_s+\sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i\right)^2}{4mb}\geq\frac{\left(a+m\left(\alpha_s-2\alpha_k\right)+\sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i\right)^2}{4m^2b}$$

from where:

$$\alpha_s\leq\min_{\substack{k=1,m\\ k\neq s}}\min\left(\frac{a\left(\sqrt{m}+1\right)-2m\alpha_k+\left(\sqrt{m}+1\right)\sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i}{m\left(\sqrt{m}-1\right)},\frac{a\left(\sqrt{m}-1\right)+2m\alpha_k+\left(\sqrt{m}-1\right)\sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i}{m\left(\sqrt{m}+1\right)}\right)$$

Therefore, the marginal cost of the leader must satisfy the condition imposed by the upper limit supαs and the expression above, otherwise the company is not leading amount or if in affirmative case doesn't have a higher maximum profit than the one of the satellites.

For two companies F_s and F_k (m=2) the condition that the maximum profit of the leader be higher than that of the satellite becomes:

$$\alpha_s\leq\min\left(\frac{a\left(\sqrt{2}+1\right)+\left(\sqrt{2}-3\right)\alpha_k}{2\left(\sqrt{2}-1\right)},\frac{a\left(\sqrt{2}-1\right)+\left(\sqrt{2}+3\right)\alpha_k}{2\left(\sqrt{2}+1\right)}\right)$$

The requirement that $\frac{a\left(\sqrt{2}+1\right)+\left(\sqrt{2}-3\right)\alpha_k}{2\left(\sqrt{2}-1\right)}\leq\frac{a\left(\sqrt{2}-1\right)+\left(\sqrt{2}+3\right)\alpha_k}{2\left(\sqrt{2}+1\right)}$ is equivalent with $a\leq\alpha_k$.

Therefore, if: $a\leq\alpha_k$ we have: $\alpha_s\leq\frac{a\left(\sqrt{2}+1\right)+\left(\sqrt{2}-3\right)\alpha_k}{2\left(\sqrt{2}-1\right)}\leq\alpha_k$ and if: $a\geq\alpha_k$ follows that: $\alpha_s\leq\frac{a\left(\sqrt{2}-1\right)+\left(\sqrt{2}+3\right)\alpha_k}{2\left(\sqrt{2}+1\right)}\leq a$.

Returning to the case of the m firms the selling price is:

$$p^*=p\left(\sum_{i=1}^{m}Q_i^*\right)=\frac{a+m\alpha_s+\sum_{\substack{i=1\\ i\neq s}}^{m}\alpha_i}{2m}>0$$

From here, it follows:

$$\Delta p^*=\frac{1}{2}\Delta\alpha_s+\frac{1}{2m}\sum_{\substack{i=1\\ i\neq s}}^{m}\Delta\alpha_i$$

Therefore, a change in the marginal cost of the leader with an amount ε, will lead to a change in price $\frac{\varepsilon}{2}$ and a change of all others will change by an amount $\frac{m-1}{2m}\varepsilon < \frac{\varepsilon}{2}$.

Returning to the situation of the two companies F_s and F_k, the condition that F_s be leader was:

$$\alpha_s < \frac{a+5\alpha_k}{6}$$

It is possible that none of the companies can not be a leader? It should, in this case that $\alpha_s \geq \frac{a+5\alpha_k}{6}$ and $\alpha_k \geq \frac{a+5\alpha_s}{6}$ that is:

$$\begin{cases} 6\alpha_s - 5\alpha_k \geq a \\ 6\alpha_k - 5\alpha_s \geq a \end{cases}$$

Considering the straight lines: $6\alpha_s - 5\alpha_k - a = 0$ and $6\alpha_k - 5\alpha_s - a = 0$ we get the situation in Figure 1:

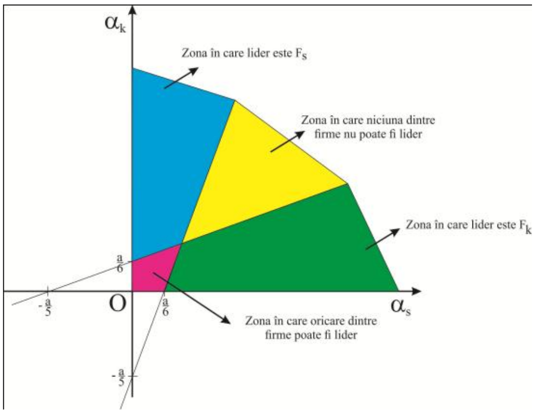

Figure 1.

Therefore, when a firm's marginal cost is sufficiently large relative to the other, it can not take the lead. If the two firms have marginal costs comparable to each other, but large enough, none can assume leadership, the reverse situation being when the marginal costs are low enough, the two companies being able to assume each leadership role.

3 References

Ioan, C.A. & Ioan, G. (2012), *Matheconomics*. Galati: Zigotto.

Ioan, C.A. & Ioan, G. (2012). General Considerations on the Oligopoly. *Acta Universitatis Danubius. Oeconomica*, Vol. 8, No. 3, pp. 115-137.

Spence, M. (1984). Cost reduction, competition, and industry performance. *Econometrica*,Vol. 52, No.1, pp. 101-121.

Van Damme, E. & S. Hurkens (1999). Endogenous Stackelberg leadership. *Games and Economic Behavior*, Vol. 28, No.1, pp. 105-129.