About the Economical Equilibrium

Catalin Angelo Ioan[1]**, Gina Ioan**[2]

Abstract: In this paper we will study the economic equilibrium problem from a general point of view with applications for polynomial demand and supply functions. We will expose and analyze also the stability of supply and demand in respect Walras and Marshall, in a general approach, detached from the usual linear. The dynamic stability after Kaldor will be presented in conjunction with fixed point theorems which provide results of great generality.

Keywords demand; supply; Walras; Marshall; Kaldor

JEL Classification: D01

1. Introduction

The economic equilibrium problem is of particular practical importance because of the dynamism of markets and the multiple influences that affect the potential stability situations. The vast majority of papers dealing with this problem in the linear case, both supply and demand functions being polynomial of degree 1. We will try in this paper, a general approach of this problem getting, we hope, interesting results that can then customize for different situations. We will expose and analyze also the stability of supply and demand in respect Walras and Marshall, in a general approach, detached from the usual linear.

The dynamic stability after Kaldor will be presented in conjunction with fixed point theorems which provide results of great generality.

[1] Associate Professor, PhD, Danubius University of Galati, Faculty of Economic Sciences, Romania, Address: 3 Galati Blvd, Galati, Romania, tel: +40372 361 102, fax: +40372 361 290, Corresponding author: catalin_angelo_ioan@univ-danubius.ro.

[2] Assistant Professor, PhD in progress, Danubius University of Galati, Faculty of Economic Sciences, Romania, Address: 3 Galati Blvd, Galati, Romania, tel: +40372 361 102, fax: +40372 361 290, e-mail: gina_ioan@univ-danubius.ro.

AUDŒ, Vol 7, no 6, pp. 129-140

2. The Economical Equilibrium

The existence of any economic activity is conditioned by two main groups of "actors": consumers and producers.

We will consider below the situation of n normal goods B_i, $i=\overline{1,n}$, produced by a number of "s" economic agents and asked by "c" buyers.

From the fact that goods are normal, at a price p_i of the good B_i, the amount requested by the j-th consumer, denoted x_{ij} will satisfy: $\frac{dx_{ij}}{dp_i} \le 0$ caeteris paribus (which the equality to zero occurs in the absence of the j-th consumer demand or if a regardless of price request). With the assumption that at least one good is sensitive at demand to price changes, considering the aggregate demand of the c consumers and noting for the good B_i: $x_i = \sum_{j=1}^{c} x_{ij}$ we will have therefore:

$$\frac{dx_i}{dp_i} = \sum_{j=1}^{c} \frac{dx_{ij}}{dp_i} < 0$$

As a result of this relationship, it appears that the aggregate demand for the good B_i will be a strictly decreasing function of its price p_i.

Also, we have that for normal goods, the demand function is convex, that is: $\frac{d^2x_i}{dp_i^2} \ge 0$.

Let therefore:

$D(p_1,...,p_n)=(x_1(p_1),...,x_n(p_n))$

the demand vector for the n goods $B_1,...,B_n$, where $x_i=x_i(p_i)$, $i=\overline{1,n}$ is the demand for the good depending on price.

Similarly, for a manufacturer, we have that for a given selling price, the optimal production for the purposes of profit maximization satisfies the relation: $Q_0=Cm^{-1}(p)$ where Cm is the marginal cost. How Cm is increasing and the inverse function has the same character as direct function, it follows that Q_0 is also an increasing function with respect to the sale price. In other words, for the producer a price increasing lead to an increased production because of market demands. Let y_{ij} denote the amount of good B_i provided by the manufacturer j. It is obvious that $\frac{dy_{ij}}{dp_i} \ge 0$ (where the equality with zero takes place in the case of a missing offering from the manufacturer j, or an offer regardless of price).

We will assume that at least a good B_i is sensitive for the purposes of supply to price changes. Considering the aggregate supply of the s producers for the good B_i and noting: $y_i=\sum_{j=1}^{s} y_{ij}$ we will have:

$$\frac{dy_i}{dp_i}=\sum_{j=1}^{s}\frac{dy_{ij}}{dp_i}>0$$

From these relations, it follows that the aggregate supply function for the good B_i is strictly increasing of its price p_i.

On the other hand, the manufacturer's offer is at the marginal cost of production, so p_i is a convex function of y_i, i.e.:

$$p_i(\lambda y_i^1+(1-\lambda) y_i^2)\le\lambda p_i(y_i^1)+(1-\lambda)p_i(y_i^2)$$

Considering the inverse function $y_i=y_i(p_i)$, as y_i is increasing, we have from the previous inequality:

$$y_i(p_i(\lambda y_i^1+(1-\lambda) y_i^2))\le y_i(\lambda p_i(y_i^1)+(1-\lambda)p_i(y_i^2))$$

or otherwise:

$$\lambda y_i(p_i^1)+(1-\lambda)y_i(p_i^2)\le y_i(\lambda p_i^1+(1-\lambda) p_i^2)$$

After these considerations, we have that y_i is a concave function, i.e.: $\frac{d^2y_i}{dp_i^2}\le 0$.

Let us note now the offer vector of the s producers:

$$S(p_1,...,p_n)=(y_1(p_1),...,y_n(p_n))$$

for the n goods $B_1,...,B_n$ where $y_i=y_i(p_i)$, $i=\overline{1,n}$ is the offer like function of price for the appropriate good.

We will say that the **market** of the n goods is in **equilibrium** if $\exists p_1,...,p_n>0$ (called **the equilibrium prices**) so that:

$$D(p_1,...,p_n)=S(p_1,...,p_n)$$

The vectorial equation of equilibrium: $D(p_1,...,p_n)=S(p_1,...,p_n)$ becomes: $x_i(p_i)-y_i(p_i)=0$. Considering the equilibrium prices $p_1,...,p_n$ will call the quantities $x_i(p_i)=y_i(p_i)$ – **amount of equilibrium**.

Before continuing, let consider the equation $x_i(p_i)=0$. Since x_i is decreasing it follows that is injective, so the equation will have at most one real root. Also, as $x_i(0)>0$ (*the maximum amount of good that can be purchased regardless of price*) and $\lim_{p_i \to \infty} x_i(p_i) < 0$ (*there is a psychological price threshold at which no longer buy anything*) will follow that $\exists p_d > 0$ such that: $x_i(p_d)=0$. Following these considerations, the analysis of the demand function related to the price will be held for $p \in [0, p_d]$.

Also, the function $y_i=y_i(p_i)$ is strictly increasing, so considering the equation $y_i(p_i)=0$ (*minimal production selling price*) let its unique solution $p_s \in \mathbf{R}$.

If $p_s \le 0$ (*in which case the manufacturer sets a minimum output*) we will put $p_s=0$. For $p_i>p_s$ we have so: $y_i(p_i)>0$ and $p_i<p_s$ will involve $y_i(p_i)<0$. The analysis of supply will be therefore for $p_i \in [p_s, \infty)$.

If $p_s>p_d$ resulting $[0,p_d] \cap [p_s,\infty)=\varnothing$ then the equilibrium cannot take place. If $p_s=p_d$ then there is no production or purchase, so again a situation of economically uninteresting.

Therefore, we will perform equilibrium analysis for $p_s<p_d$ and $p_i \in (p_s, p_d)$.

Noting now $E_i(p_i)=x_i(p_i)-y_i(p_i)$ we have: $\frac{dE_i}{dp_i} = \frac{dx_i}{dp_i} - \frac{dy_i}{dp_i} < 0$, $i=\overline{1,n}$ hence the equation $E_i(p_i)=0$ having at most one real root. Let $E_i(p_s)=x_i(p_s)-y_i(p_s)$ and $E_i(p_d)=x_i(p_d)-y_i(p_d)$. From the fact that the function E_i is strictly decreasing, it follows: $E_i(p_s)>E_i(p_d)$. As a result, it follows:

- $E_i(p_s) \le 0$ implies lack of equilibrium price for the good B_i;
- $E_i(p_s)>0$ and $E_i(p_d)>0$ implies also the absence of equilibrium price for the good B_i;
- $E_i(p_s)>0$ and $E_i(p_d)<0$ implies a unique equilibrium price for the good B_i.

As an example, consider the aggregate demand non-null and non-constant for a good B_i of polynomial type:

$$x_i = \sum_{k=0}^{r_1} a_{ik} p_i^k = a_{i0} + a_{i1} p_i + \ldots + a_{ir_1} p_i^{r_1},\ r_1 \ge 1$$

and analog, the aggregate supply non-null and non-constant for the same good:

$$y_i = \sum_{k=0}^{r_2} b_{ik} p_i^k = b_{i0} + b_{i1} p_i + \ldots + b_{ir_2} p_i^{r_2},\ r_2 \ge 1$$

The condition that $\frac{dx_i}{dp_i} < 0$ is provided at: $d_1(p_i) = \sum_{k=1}^{r_1} k a_{ik} p_i^{k-1} < 0\ \forall p_i \in (p_s, p_d)$.

Also, the convexity of demand function implies: $d_2(p_i)=\sum_{k=2}^{r_1} k(k-1)a_{ik}p_i^{k-2} \geq 0$ $\forall p_i \in (p_s,p_d)$.

Similarly, for the supply, the condition that $\frac{dy_i}{dp_i}>0$ becomes: $s_1(p_i)=\sum_{k=1}^{r_2} kb_{ik}p_i^{k-1}>0$ $\forall p_i \in (p_s,p_d)$ and the concavity: $s_2(p)=\sum_{k=2}^{r_2} k(k-1)b_{ik}p_i^{k-2} \leq 0$ $\forall p_i \in (p_s,p_d)$.

The equation for determining the equilibrium price for the good B_i is:

$$\sum_{k=0}^{r_1} a_{ik}p_i^k - \sum_{k=0}^{r_2} b_{ik}p_i^k = 0,\ p_i \in (p_s,p_d)$$

The unique price equilibrium condition becomes, from above to:

$$\left(\sum_{k=0}^{r_1} a_{ik}p_s^k - \sum_{k=0}^{r_2} b_{ik}p_s^k\right)\left(\sum_{k=0}^{r_1} a_{ik}p_d^k - \sum_{k=0}^{r_2} b_{ik}p_d^k\right)<0$$

In the particular case of linear demand and supply, let: $x_i=a-bp_i$ and $y_i=c+dp_i$, $a,b,d>0$, $c \in \mathbf{R}$. The equation $x_i(p_i)=0$ implies $p_d=\frac{a}{b}$ and $y_i(p_i)=0$: $p_s=-\frac{c}{d}$.

If $c \geq 0$ then $p_s=0$. We will consider so $p_s=\max\left\{-\frac{c}{d},0\right\}$. The problem of determining the equilibrium price is so meaningful for $p_s<p_d \Leftrightarrow \max\left\{-\frac{c}{d},0\right\}<\frac{a}{b}$.

If $c \geq 0$ then the inequality becomes: $\frac{a}{b}>0$ which is true, and if $c<0$ then: $-\frac{c}{d}<\frac{a}{b}$ $\Leftrightarrow ad+bc>0$.

We also have: $\frac{dx_i}{dp_i}=-b<0$, $\frac{d^2x_i}{dp_i^2}=0$, $\frac{dy_i}{dp_i}=d>0$, $\frac{d^2y_i}{dp_i^2}=0 \leq 0$ then the problem conditions are met.

The unique price equilibrium condition is, from above, to:

$(a-bp_s-c-dp_s)(a-bp_d-c-dp_d)<0$

For $c \geq 0$ the condition reduces to:

(a-c)$\left(a-b\frac{a}{b}-c-d\frac{a}{b}\right)$ <0⇔-(a-c) $\frac{ad+bc}{b}$ <0⇔c<a

If c<0 then:

$$\left(a+b\frac{c}{d}-c+d\frac{c}{d}\right)\left(a-b\frac{a}{b}-c-d\frac{a}{b}\right)<0\Leftrightarrow -\frac{(ad+bc)^2}{bd}<0 - \text{true}$$

From the equation $x_i=y_i$ follows: $a-bp_i=c+dp_i$ therefore:

$$\overline{p}_i=\frac{a-c}{b+d},\ \overline{x}_i=\overline{y}_i=\frac{ad+bc}{b+d}$$

If a<c then we have seen above that the equilibrium between demand and supply for the good B_i cannot take place.

3. The Equilibrium Behavior to Changes in Demand or Supply

Consider, as above, a number of goods $B_1,...,B_n$ and $p_1,...,p_n$ the appropriate prices. Let also the aggregate demand functions: $x_i=x_i(p_i)$, $i=\overline{1,n}$ and the supply functions: $y_i=y_i(p_i)$, $i=\overline{1,n}$. Where all goods are normal, we saw that: $\frac{dx_i}{dp_i}<0$ and, also: $\frac{dy_i}{dp_i}>0$, $i=\overline{1,n}$.

For a fixed good B_i, in the equilibrium state, the price p_i satisfy the equality: $x_i(p_i)=y_i(p_i)$ in the additional conditions: $E_i(p_s)>0$ and $E_i(p_d)<0$.

Because the functions x_i and y_i are strictly monotonous it follows that they are invertible on a restriction of their co-domain, so we can write:

$p_i=f_i(x_i)$ with $f_i(x_i(p_i))=p_i$ and $x_i(f_i(x_i^*))=x_i^*$

the function f_i being (*by virtue of preserving the monotony at invertability*) strictly decreasing and analogously:

$p_i=g_i(y_i)$ with $g_i(y_i(p_i))=p_i$ and $y_i(g_i(y_i^*))=y_i^*$

the function g_i being (*for the same reasons*) strictly increasing.

3.1. The Demand Change – the Walras Stability

Let now consider a change in the demand from the equilibrium state from $\left(p_i^*,x_i^*,y_i^*\right)$ at $\overline{x}_i^*$.

First, we assume that $\overline{x}_i^* > x_i^*$. In order to meet demand, at equilibrium, will be satisfied the condition that: $y_i(p_i) = \overline{x}_i^* > x_i^*$.

Because the function y_i is strictly increasing, the solution of the equation will be $\overline{p}_i^* > p_i^*$. On the other hand, as the demand function is strictly decreasing in relation to the price, we have: $\overline{\overline{x}}_i^* = x_i(\overline{p}_i^*) < x_i(p_i^*) = x_i^*$ therefore a contradiction in relation to the additional demand for product. For this reason, the manufacturer will not fully meet the additional demand and provide $x_i^{**} \in \left(x_i^*, \overline{x}_i^*\right)$. How g_i is strictly increasing, from the inequality: $x_i^* < x_i^{**} < \overline{x}_i^*$ follows $g(x_i^*) < g(x_i^{**}) < g(\overline{x}_i^*)$ therefore: $p_i^* < p_i^{**} < \overline{p}_i^*$.

Following these considerations, it appears that on additional demand for goods, the manufacturer will provide only part of them at a higher price than in the original equilibrium.

Suppose now that $\overline{x}_i^* < x_i^*$ therefore the buyers require a smaller number of products. In order to meet demand, at equilibrium, will be satisfied the condition that: $y_i(p_i) = \overline{x}_i^* < x_i^*$.

Because the function y_i is strictly increasing, the solution of the equation will be $\overline{p}_i^* < p_i^*$. As the demand function is strictly decreasing in relation to price, we have: $\overline{\overline{x}}_i^* = x_i(\overline{p}_i^*) > x_i(p_i^*) = x_i^*$ then a contradiction in relation to reduced product demand. For this reason, the manufacturer will not reduce its offer completely (*corresponding to decreasing demand*) and will provide $x_i^{**} \in \left(\overline{x}_i^*, x_i^*\right)$. How g_i is strictly increasing, from the inequality: $\overline{x}_i^* < x_i^{**} < x_i^*$ follows that: $g_i(\overline{x}_i^*) < g_i(x_i^{**}) < g_i(x_i^*)$ or, with other words: $\overline{p}_i^* < p_i^{**} < p_i^*$. After these considerations, it follows that on a decreasing demand for goods, the manufacturer will reduce only some of them at a price lower than at the initial equilibrium.

3.2. The Demand Change – the Marshall Stability

Let now consider a change in the price from the equilibrium state from $\left(p_i^*, x_i^*, y_i^*\right)$ at $\overline{p}_i^*$.

First, we assume that $\overline{p}_i^* > p_i^*$. Since the demand function is strictly increasing, we have: $\overline{y}_i^* = y_i(\overline{p}_i^*) > y_i(p_i^*) = y_i^* = x_i^*$. On the other hand, the function f_i being strictly decreasing, follows: $\overline{\overline{p}}_i^* = f_i(\overline{y}_i^*) < f_i(x_i^*) = p_i^*$. The asking price becomes lower than the initial which in a contradiction with the assumption. Due to this, the producer will not increase as much the price of the good as he wishes, but will choose a price $p_i^{**} \in (p_i^*, \overline{p}_i^*)$.

From the inequality $p_i^* < p_i^{**} < \overline{p}_i^*$ we have: $f_i(p_i^*) > f_i(p_i^{**}) > f_i(\overline{p}_i^*)$ therefore: $x_i^* > x_i^{**} > \overline{x}_i^*$. Following these considerations, it follows that after an increasing of price, the manufacturer will provide only some additional product at a price higher than at the equilibrium, but lower than the one want.

If now $\overline{p}_i^* < p_i^*$, because the supply function is strictly increasing, we have: $\overline{y}_i^* = y_i(\overline{p}_i^*) < y_i(p_i^*) = y_i^* = x_i^*$.Because the function f_i is strictly decreasing, follows: $\overline{\overline{p}}_i^* = f_i(\overline{y}_i^*) > f_i(x_i^*) = p_i^*$. After these facts, the asking price becomes higher than the originally, contrary to the assumption made. Due to this, the manufacturer will not diminish the good price as much as he wants, but will choose a price $p_i^{**} \in (\overline{p}_i^*, p_i^*)$.

After the inequality $\overline{p}_i^* < p_i^{**} < p_i^*$ we have: $f_i(\overline{p}_i^*) > f_i(p_i^{**}) > f_i(p_i^*)$ therefore: $\overline{x}_i^* > x_i^{**} > x_i^*$. We got so that after reducing the price, the manufacturer will increase the quantity of products offered, but to a lesser extent than market demand declined in price.

3.3. The Supply Change – the Walras Stability

Let now a change in the supply from those at equilibrium (p_i^*, x_i^*, y_i^*) to $\overline{y}_i^*$.

We will assume initially that $\overline{y}_i^* > y_i^*$. The added supply from the manufacturer entails, at equilibrium, the new price will satisfy the condition: $x_i(p_i) = \overline{y}_i^* > y_i^*$. Because the function x_i is strictly decreasing, the solution of the equation will be $\overline{p}_i^* < p_i^*$. On the other hand, as supply is a strictly increasing function with respect to price, we have: $\overline{\overline{y}}_i^* = y_i(\overline{p}_i^*) < y_i(p_i^*) = y_i^*$ therefore a contradiction in relation to supply additional product. For this reason, the manufacturer will not provide all the additional amount and will reduce it to $y_i^{**} \in (y_i^*, \overline{y}_i^*)$. How f_i is strictly decreasing, from the inequality: $y_i^* < y_i^{**} < \overline{y}_i^*$ follows that $f_i(y_i^*) > f_i(y_i^{**}) > f_i(\overline{y}_i^*)$ so: $p_i^* > p_i^{**} > \overline{p}_i^*$. Following these considerations, we get that after an additional supply of goods, the manufacturer will have to reduce the selling price from the equilibrium state.

Suppose now that $\overline{y}_i^* < y_i^*$ therefore the manufacturer offers a smaller number of products. The supply decrease entails, at equilibrium, the new price that satisfies the condition: $x_i(p_i)=\overline{y}_i^* < y_i^*$. Because the function x_i is strictly decreasing, the solution of the equation will be $\overline{p}_i^* > p_i^*$. Because the supply function is strictly increasing with the price, we have: $\overline{\overline{y}}_i^*=y_i(\overline{p}_i^*)>y_i(p_i^*)=y_i^*$ therefore a contradiction in relation to the diminished supply of product. For this reason, the manufacturer will reduce its offer and provide $y_i^{**} \in (\overline{y}_i^*, y_i^*)$. How f_i is strictly decreasing, from the inequality: $\overline{y}_i^* < y_i^{**} < y_i^*$ follows: $f_i(\overline{y}_i^*)>f_i(y_i^{**})>f_i(y_i^*)$ therefore: $\overline{p}_i^* > p_i^{**} > p_i^*$. Following these considerations, it follows that after decreasing the supply of goods, the manufacturer will increase the selling price to the initial at equilibrium.

3.4. The Supply Change – the Marshall Stability

Now, let consider a change in price from (p_i^*, x_i^*, y_i^*) at $\overline{p}_i^*$.

First we assume that $\overline{p}_i^* > p_i^*$. Since the demand function is strictly decreasing, we have: $\overline{x}_i^*=x_i(\overline{p}_i^*)<x_i(p_i^*)=x_i^*=y_i^*$. On the other hand, the function g_i being strictly increasing, it follows: $\overline{\overline{p}}_i^*=g_i(\overline{x}_i^*)<g_i(y_i^*)=p_i^*$. The asking price is lower than initially in contradiction with the assumption. Due to this, the producer will not increase the price as much as he wishes, but will choose a price $p_i^{**} \in (p_i^*, \overline{p}_i^*)$. After the inequality: $p_i^* < p_i^{**} < \overline{p}_i^*$ follows: $g_i(p_i^*)<g_i(p_i^{**})<g_i(\overline{p}_i^*)$ therefore: $y_i^* < y_i^{**} < \overline{y}_i^*$. Following these considerations, we have that at an increase of the price, the manufacturer will provide only some additional product at a price higher than the equilibrium, but lower than the one he want.

If now $\overline{p}_i^* < p_i^*$, because the demand is a strictly decreasing function, we have: $\overline{x}_i^*=x_i(\overline{p}_i^*)>x_i(p_i^*)=x_i^*=y_i^*$. The function g_i is strictly increasing and we have: $\overline{\overline{p}}_i^*=g_i(\overline{x}_i^*)>g_i(y_i^*)=p_i^*$. Therefore, the asking price becomes higher than the original, contrary to the assumption made. Due to this, the manufacturer will not diminish the good price as long as he wants, but will choose a price $p_i^{**} \in (\overline{p}_i^*, p_i^*)$. After the inequality $\overline{p}_i^* < p_i^{**} < p_i^*$ follows: $g_i(\overline{p}_i^*)<g_i(p_i^{**})<g_i(p_i^*)$ and $\overline{y}_i^* < y_i^{**} < y_i^*$.

We have got so that after reducing the price, the manufacturer will increase the quantity of products offered, but to a lesser extent than market demand declined in price.

3.5. The Kaldor Dynamic Stability

After the analysis performed above, we noticed that the demand and supply adjust each other in the meaning that a change in the amount of good offered entails balancing price change, as any price change involves a readjustment of the amount of balance. Unfortunately, this process does not occur instantaneously recalculation, but by successive tappings of the two parties.

Before starting the actual analysis, let consider the demand function for a normal good of the form $x=x(p)$ with $\frac{dx}{dp}<0$ and the supply function: $y=y(p)$ with $\frac{dy}{dp}>0$.

Let consider therefore a starting time $t_0=0$ and a price p_0 offered by the manufacturer for a good B_i. The supply function will therefore be: $y_0=y(p_0)$. At time $t_1=1$, the buyer will be willing to offer that price p_1 (*to establish the equilibrium*) for which: $x_1=x(p_1)=y(p_0)=y_0$. On the other hand, the manufacturer will provide the buyer at the price requested, the quantity $y_1=y(p_1)$. Proceeding similarly, at the time t_n, the price offered by the buyer will be: $x_n=x(p_n)=y(p_{n-1})$ and the amount offered by the manufacturer: $y_n=y(p_n)$.

The readjustment process thus involves the following steps sequentially:

$y_0=y(p_0)$
$p_1=x^{-1}(y_0)$
$y_1=y(p_1)$
...
$p_n=x^{-1}(y_{n-1})$
$y_n=y(p_n)$
$p_{n+1}=x^{-1}(y_n)$
...
The question arises is the determining the equilibrium price p^* if it is exists.

From the above equalities, follow the recurrence relation:

$p_{n+1}=x^{-1}(y(p_n))\ \forall n\geq 0$

The function $z=x^{-1}\circ y$ is strictly decreasing because $\frac{dz}{dp}=\frac{\frac{dy}{dp}}{\frac{dx}{dp}}<0$. But y being continuous, moving to limit in the recurrence relation above, follows: $p^*=z(p^*)$ therefore the equilibrium price p^* is a fixed point of the function $z=x^{-1}\circ y$.

The existence of a fixed point for the function z, requires some additional conditions.

We will say that a function $f:(a,b)\to\mathbf{R}$, $a<b\in\mathbf{R}$, is called a contraction application if $\exists C\in(0,1)$ such that: $\forall x,y\in\mathbf{R}$: $|f(x)-f(y)|\leq C\cdot|x-y|$.

Also, a function $f:(a,b)\to\mathbf{R}$, $a<b\in\mathbf{R}$, is called a Lipschitz application if $\exists L>0$ such that: $\forall x,y\in\mathbf{R}$: $|f(x)-f(y)|\leq L\cdot|x-y|$.

It is noted that a contraction application meets the requirement of Lipschitz. Also, any function of class C^1 such that $|f'(x)|<1$ $\forall x\in(a,b)$ from the Lagrange's theorem is a contraction application.

Any application of contraction has at most one fixed point. Indeed, if $\exists x^*\neq y^*\in\mathbf{R}$ such that $f(x^*)=x^*$ and $f(y^*)=y^*$ then: $|x^*-y^*|=|f(x^*)-f(y^*)|\leq C\cdot|x^*-y^*|$ therefore $C\geq 1$ – contradiction comes from the assumption that $x^*\neq y^*$.

Considering now a fixed point $x^*\in\mathbf{R}$ and an initial value x_0, let the range $x_n=f(x_{n-1})$ $\forall n\geq 1$. We therefore have:

$$|x_n-x^*|=|f(x_{n-1})-f(x^*)|\leq C\cdot|x_{n-1}-x^*|\leq\ldots\leq C^n\cdot|x_0-x^*|$$

Passing to the limit as $n\to\infty$ follows: $\lim x_n=x^*$ therefore the range converges to the fixed point.

Also from Banach fixed point theorem, we have that any application of contraction admits a fixed point. Following these considerations, to have guaranteed the existence of price equilibrium, we will impose the additional condition that the application $z=x^{-1}\circ y$ must satisfy $|z'(p)|<1$ $\forall p>0$.

In particular, for linear functions of supply and demand, we have:

$x(p)=a-bp$, $y(p)=c+dp$, $a,b,d>0$, $c<a$

from where:

$$z(p)=x^{-1}(y(p))=\frac{a-(c+dp)}{b}=\frac{a-c-dp}{b}$$

The equilibrium price is a fixed point of the function z, so we have:

$$\frac{a-c-dp^*}{b}=p^*$$

from where:

$$p^*=\frac{a-c}{b+d}$$

We have $z'(p) = -\frac{d}{b}$ therefore the condition $\frac{d}{b} < 1$ or otherwise $d < b$ guarantees the existence of price equilibrium.

4. Conclusion

From the above, a first significant fact is that the economic equilibrium cannot occur under all conditions, being influenced primarily by prices charged by the buyer, followed by the manufacturers.

Another important result derived from the above analysis is to formulate the equilibrium conditions when supply and demand are of polynomial nature that extends the usual linear analysis.

The Walras or Marshall stability of the change in demand or supply has been treated for general functions, highlighting significant aspects of behavior change.

The dynamic stability after Kaldor has been broach using the fixed point theorem of Banach, extending the classical approach to a very general class of situations.

5. References

Chiang A.C. (1984). *Fundamental Methods of Mathematical Economics*. McGraw-Hill Inc.

Stancu S. (2006). *Microeconomics*. Bucharest: Economica.

Varian H.R. (2006). *Intermediate Microeconomics*. W.W. Norton & Co.